Influence of Area Source Shape and Orientation on the Spatial Variation of Emissions

John E. Stout
Wind Erosion Research Unit
USDA-Agricultural Research Service
Lubbock, Texas 79401

Abstract

An area source is assumed to emit gases or fine aerosols into the air above at a steady and uniform rate and the emissions are then advected downstream by the ambient wind. Analytical solutions are obtained for the variation of horizontal mass flow across rectangular and elliptical area sources. The rate at which the total mass flow increases with horizontal distance across the source is found to be a direct function of the source shape, the wind angle, and the aspect ratio of the source.

Introduction

As the wind sweeps across the surface of the earth it often encounters well-defined regions that provide an area source of gases or particulate matter. Emissions are transported by the wind across the source and develop into an increasingly concentrated low level plume of advected material. Plumes originating from area sources such as landfills, waste containment ponds, or cattle feedlots often contain noxious fumes. Wind-swept agricultural lands, construction sites, or dry lake beds may spawn plumes of fine dust particles. Area sources can also act as exchange regions for the natural transfer of beneficial gases and aerosols. For example, a vegetated region can release oxygen to the atmosphere and water vapor can be transferred from the surface of a lake to the air above. Whether the release is a natural environmental process or an environmental catastrophe the governing physical process is the same.

There have been major efforts to model area-source releases to the atmosphere (Crane *et al.*, 1977; Petersen, 1978; Snyder, 1991; Chrysikopoulos *et al.*, 1992; EPA, 1992a; Schmid, 1994). It has been pointed out that current models such as the Industrial Source Complex Dispersion Model (EPA, 1992b) or the Point, Area and Line Source Model (Petersen, 1978) yield unrealistic predictions for receptors located within or nearby the area source (EPA, 1989). In such models area sources are modeled as numerous discrete point sources distributed within the source area; the collective output of all the distributed point sources being equal to the output of an equivalent area source. This type of modeling approach is sufficient when the receptor point is far downwind of the source, however, it fails to provide a proper description of the concentration field within or near source boundaries.

This paper represents an attempt to better describe the emissions upwind of a point within the boundaries of an area source by considering the influence of source shape and source orientation with respect to the wind. We further calculate the advection of emissions across the source. This effort represents a first step toward an improved area source model. Much work remains before these tentative steps yield a useful dispersion model for area sources.

Theory

Notation for flow across a flat rectangular and elliptical area source may be reduced to that shown in Figs. 1 and 6, respectively. Here, it is convenient to define a wind-relative coordinate system, denoted (x,y), and a body-fixed coordinate system, denoted (x',y'). In the wind-relative coordinate system the x-axis is aligned with the wind direction and the origin is positioned at the most extreme windward edge of the area source. Thus, both the origin and the orientation of the wind-relative coordinates vary with wind direction. On the other hand, the origin of the body-fixed coordinate system is placed at the center of the area source and the axes remain fixed to the source.

The maximum fetch L across an area source is defined as the distance between the most upwind point to the most downwind point measured along a line parallel to the wind direction. The size and shape of the area source as well as the wind angle influences the value of L. The maximum fetch L provides a convenient length scale, since in dimensionless form the area source always extends from $0 < x/L < 1$, regardless of the source shape and orientation.

The following analysis assumes that the flow of material from the source is uniform within the boundaries of the source and zero outside the source area. It is assumed that there is no deposition of the introduced material. With these restrictions it is possible to explore exclusively the effects of area source geometry and wind angle without involving other factors.

The local components of mass flux in the horizontal, lateral, and vertical direction are denoted by $f_x(x,y,z,t)$, $f_y(x,y,z,t)$, and $f_z(x,y,z,t)$, respectively. Where mass flux is the product of the local concentration, $c(x,y,z,t)$, and the appropriate component of particle velocity. In differential form the mass conservation condition may be expressed as:

$$\frac{\partial c}{\partial t} + \frac{\partial f_x}{\partial x} + \frac{\partial f_y}{\partial y} + \frac{\partial f_z}{\partial z} = 0 \ . \qquad (1)$$

Assuming a steady ambient flow, and integrating Eq. (1) over a vertical plane bounded at the surface yields

$$\frac{\partial}{\partial x}\int_0^\infty\int_{-\infty}^\infty f_x\, dy\, dz + \frac{\partial}{\partial y}\int_0^\infty\int_{-\infty}^\infty f_y\, dy\, dz + \frac{\partial}{\partial z}\int_0^\infty\int_{-\infty}^\infty f_z\, dy\, dz = 0 \; . \tag{2}$$

The first double integral of Eq. (2) is simply an expression for the total horizontal mass flow, $Q(x)$, where

$$Q(x) = \int_0^\infty\int_{-\infty}^\infty f_x(x,y,z)\, dy\, dz \; . \tag{3}$$

Note that $Q(x)$ is expressed in units of mass per unit time.

By changing the order of integration and differentiation, and considering the lateral boundary conditions $f_y(x,4,z) = f_y(x,-4,z) = 0$, the second term may be eliminated as follows:

$$\frac{\partial}{\partial y}\int_0^\infty\int_{-\infty}^\infty f_y\, dy\, dz = \int_0^\infty\int_{-\infty}^\infty \frac{\partial f_y}{\partial y}\, dy\, dz$$

$$= \int_0^\infty \left[f_y(x,\infty,z) - f_y(x,-\infty,z)\right] dz = 0 \; . \tag{4}$$

Vertical flux at the air/source interface is assumed constant with a value of $f_z(x,y,0) = f_{zo}$ within the source area and is assumed to be zero outside the source area. Far above the surface the vertical flux is zero, $f_z(x,y,4) = 0$, thus, the third term reduces to

$$\frac{\partial}{\partial z}\int_0^\infty\int_{-\infty}^\infty f_z\, dy\, dz = \int_{-\infty}^\infty\int_0^\infty \frac{\partial f_z}{\partial z}\, dz\, dy$$

$$= \int_{-\infty}^\infty \left[f_z(x,y,\infty) - f_z(x,y,0)\right] dy = -S(x) f_{zo} \; , \tag{5}$$

where $S(x)$ is the crosswind width of the source at any given downwind distance, x. Thus, Eq. (2) may be rewritten as:

$$\frac{\partial Q}{\partial x} = S(x)\, f_{zo} \; , \tag{6}$$

which states that the gradient of the mass flow in the x-direction equals the product of the local vertical surface flux times the crosswind width of the source.

Since it is assumed that f_{zo} is constant at any point on the source surface, the variation of $Q(x)$ depends only on $S(x)$, which, in turn, depends on the shape of the source and its orientation with respect to the wind. Thus, the problem of determining the variation of $Q(x)$ reduces to a problem of geometry involving the crosswind width function, $S(x)$. We need only define analytically the width function then integrate Eq. (6) to find $Q(x)$.

Square or Rectangular Sources

Equation (6) could be applied to any area source shape. Here we consider a rectangular area source with smaller side **a** and larger side **b**, oriented at an angle α_w to the wind, as shown in Fig. 1.

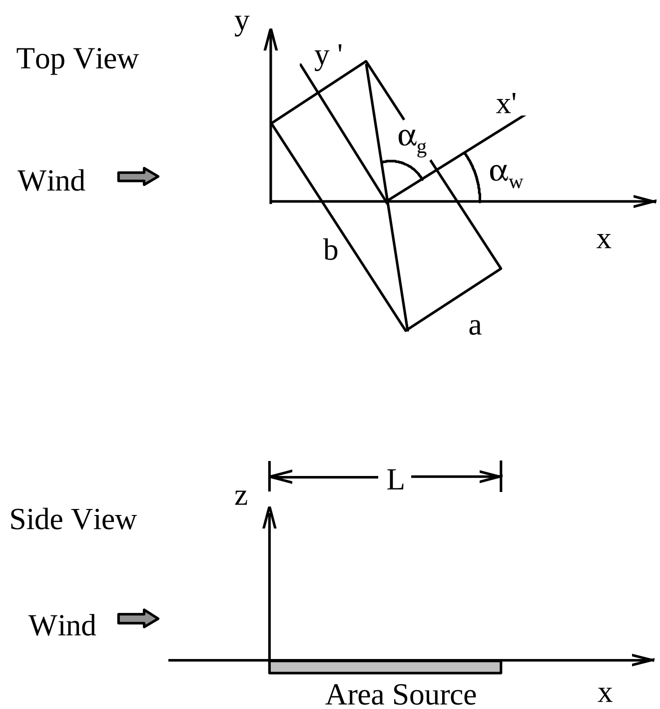

Figure 1. Definition sketch for a rectangular area source.

The aspect ratio ε is defined as the ratio **b/a**. Thus, a square has an aspect ratio of one and rectangles have aspect ratio greater than one. As shown in Fig. 1, the geometric angle α_g is defined as the angle between the diagonal and the small side **a** and may be expressed as a function of the aspect ratio as

$$\alpha_g = \tan^{-1}(\varepsilon) \; . \tag{7}$$

The maximum fetch, $L(\alpha_w)$, for a given wind angle α_w may be expressed as:

$$L(\alpha_w)/a = |\cos\alpha_w| + \varepsilon\, |\sin\alpha_w| \; . \tag{8}$$

Figure 2 shows $\boldsymbol{L(\alpha_w)}$ plotted for a square with aspect ratio 1 and for rectangles with aspect ratio 2 and 3. Note that the maximum fetch can never be less than the smallest side **a** nor can it exceed the length of the diagonal.

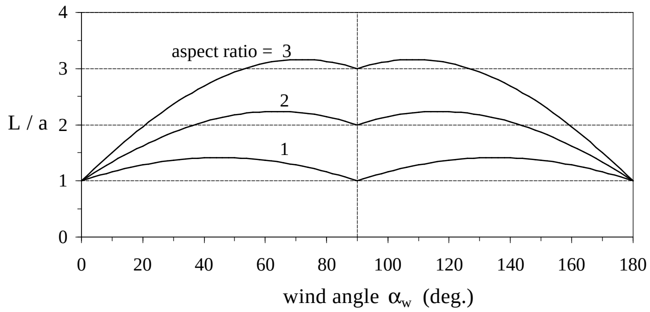

Figure 2. Maximum fetch L for rectangular area sources of various aspect ratio and wind angle.

The crosswind width function, ***S(x)***, for a rectangle is not continuous at all wind angles. There are generally three distinct regions denoted by roman numerals in Fig. 3. In region (I), ***S(x)*** increases linearly with ***x***. In region (II), ***S(x)*** is constant. In region (III), ***S(x)*** decreases linearly with ***x***.

For $\alpha_w = 0$ or $\pi/2$, regions (I) and (III) are absent and ***S(x)*** is constant for $0 < x/L < 1$. For $\alpha_w = \pi/2 - \alpha_g$, region (II) is absent, thus, ***S(x)*** increases linearly in region (I) then immediately decreases linearly in region (III). For all other values of α_w all three regions exist.

For small wind angles, $0 < \alpha_w < \pi/2 - \alpha_g$, the extent of region (I) is determined by the lower left corner of the rectangle and the border between regions (II) and (III) is determined by the upper right hand corner of the rectangle. For large wind angles, $\pi/2 - \alpha_g < \alpha_w < \pi/2$, the extent of region (I) is determined by the upper right corner of the rectangle and the border between regions (II) and (III) is determined by the lower left corner of the rectangle. Thus, for this reason and those mentioned in the previous paragraph, it is necessary to define ***S(x)*** separately for α_w greater than or less than $\pi/2 - \alpha_g$.

The cross wind width, ***S(x)***, can be determined as a function of α_w and ε for both small and large wind angles. The results are presented in Eqs. (9) through (20).

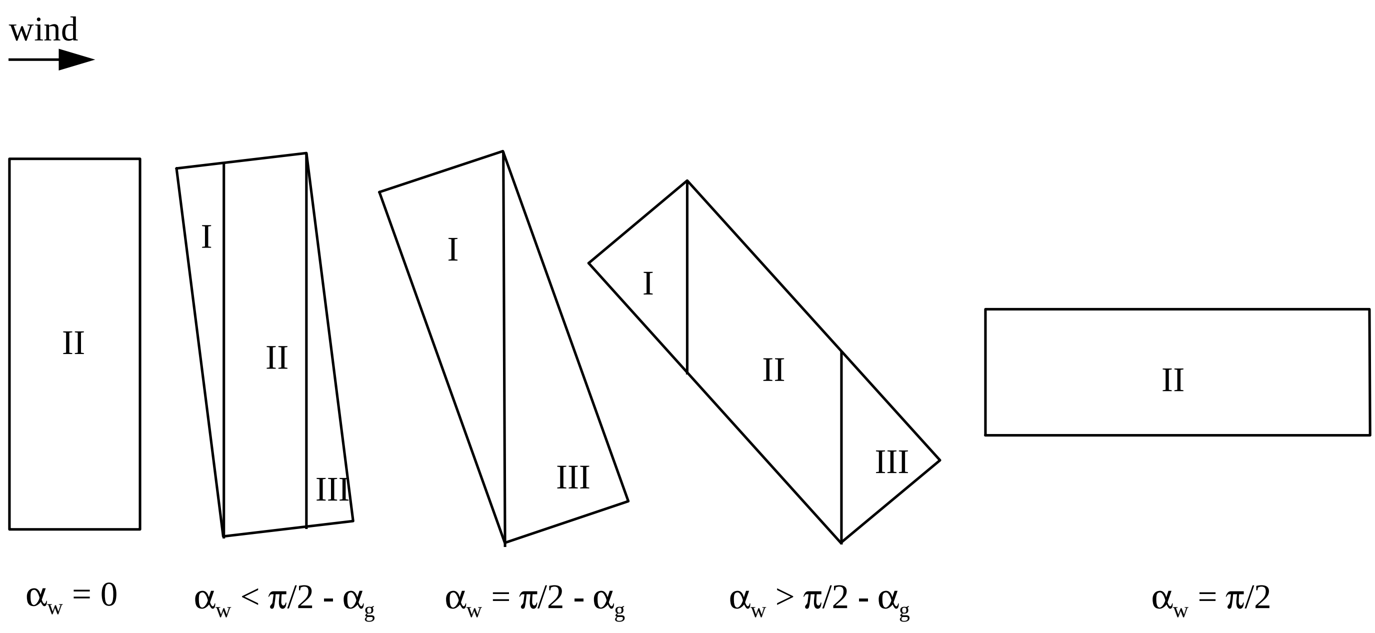

Figure 3. Regions within the area source where the crosswind width *S(x)* increases (I), remains constant (II), and decreases (III) with distance *x*.

Crosswind width for small wind angles, $0 < \alpha_w \# \pi/2 - \alpha_g$

Region (I):

$$0 < \frac{x}{L} < \frac{1}{1 + (\cot \alpha_w)/\varepsilon} \tag{9}$$

$$\frac{S_I(x)}{L} = [\tan \alpha_w + \cot \alpha_w] \frac{x}{L} \tag{10}$$

Region (II):

$$\frac{1}{1 + (\cot \alpha_w)/\varepsilon} < \frac{x}{L} < \frac{1}{1 + \varepsilon \tan \alpha_w} \tag{11}$$

$$\frac{S_{II}(x)}{L} = \frac{\tan \alpha_w + \cot \alpha_w}{1 + (\cot \alpha_w)/\varepsilon} \tag{12}$$

Region (III):

$$\frac{1}{1 + \varepsilon \tan \alpha_w} < \frac{x}{L} < 1 \tag{13}$$

$$\frac{S_{III}(x)}{L} = [\tan \alpha_w + \cot \alpha_w] \left(1 - \frac{x}{L}\right) \tag{14}$$

Crosswind width for large wind angles, $\pi/2 - \alpha_g < \alpha_w < \pi/2$

Region (I):

$$0 < \frac{x}{L} < \frac{1}{1 + \varepsilon \tan \alpha_w} \tag{15}$$

$$\frac{S_I(x)}{L} = [\tan \alpha_w + \cot \alpha_w] \frac{x}{L} \tag{16}$$

Region (II):

$$\frac{1}{1 + \varepsilon \tan \alpha_w} < \frac{x}{L} < \frac{1}{1 + (\cot \alpha_w)/\varepsilon} \tag{17}$$

$$\frac{S_{II}(x)}{L} = \frac{\tan \alpha_w + \cot \alpha_w}{1 + \varepsilon \tan \alpha_w} \tag{18}$$

Region (III):

$$\frac{1}{1 + (\cot \alpha_w)/\varepsilon} < \frac{x}{L} < 1 \tag{19}$$

$$\frac{S_{III}(x)}{L} = [\tan \alpha_w + \cot \alpha_w] \left(1 - \frac{x}{L}\right) \tag{20}$$

Solutions for Advected Mass Flow *Q(x)*

Once the crosswind width, ***S(x)***, is known then Eq. (6) may be integrated with respect to ***x*** within each region and matched at each region boundary to obtain the solutions for the advected mass flow ***Q(x)***.

Solution for small wind angles, $0 < \alpha_w$ # $\pi/2 - \alpha_g$

Region (I):

$$0 < \frac{x}{L} < \frac{1}{1 + (\cot \alpha_w)/\varepsilon} \tag{21}$$

$$\frac{Q_I(x)}{Q_{\max}} = \left(1 + \frac{1}{2} \left[\varepsilon \tan \alpha_w + \frac{\cot \alpha_w}{\varepsilon} \right] \right) \left(\frac{x}{L} \right)^2 \tag{22}$$

Region (II):

$$\frac{1}{1 + (\cot \alpha_w)/\varepsilon} < \frac{x}{L} < \frac{1}{1 + \varepsilon \tan \alpha_w} \tag{23}$$

$$\frac{Q_{II}(x)}{Q_{\max}} = \left(1 + \varepsilon \tan \alpha_w \right) \frac{x}{L} - \frac{\varepsilon}{2} \tan \alpha_w \tag{24}$$

Region (III):

$$\frac{1}{1 + \varepsilon \tan \alpha_w} < \frac{x}{L} < 1 \tag{25}$$

$$\frac{Q_{III}(x)}{Q_{\max}} = \left(1 + \frac{1}{2} \left[\varepsilon \tan \alpha_w + \frac{\cot \alpha_w}{\varepsilon} \right] \right) \left(2 - \frac{x}{L} \right) \frac{x}{L} - \frac{1}{2} \left(\varepsilon \tan \alpha_w + (\cot \alpha_w)/\varepsilon \right) \tag{26}$$

Solution for large wind angles, $\pi/2 - \alpha_g < \alpha_w < \pi/2$

Region (I):

$$0 < \frac{x}{L} < \frac{1}{1 + \varepsilon \tan \alpha_w} \tag{27}$$

$$\frac{Q_I(x)}{Q_{\max}} = \left(1 + \frac{1}{2} \left[\varepsilon \tan \alpha_w + \frac{\cot \alpha_w}{\varepsilon} \right] \right) \left(\frac{x}{L} \right)^2 \tag{28}$$

Region (II):

$$\frac{1}{1 + \varepsilon \tan \alpha_w} < \frac{x}{L} < \frac{1}{1 + (\cot \alpha_w)/\varepsilon} \tag{29}$$

$$\frac{Q_{II}(x)}{Q_{\max}} = \left(1 + \frac{\cot \alpha_w}{\varepsilon} \right) \frac{x}{L} - \frac{\cot \alpha_w}{2\varepsilon} \tag{30}$$

Region (III):

$$\frac{1}{1 + (\cot \alpha_w)/\varepsilon} < \frac{x}{L} < 1 \tag{31}$$

$$\frac{Q_{III}(x)}{Q_{\max}} = \left(1 + \frac{1}{2} \left[\varepsilon \tan \alpha_w + \frac{\cot \alpha_w}{\varepsilon} \right] \right) \left(2 - \frac{x}{L} \right) \frac{x}{L} - \frac{1}{2} \left(\varepsilon \tan \alpha_w + (\cot \alpha_w)/\varepsilon \right) \tag{32}$$

Sample curves are plotted in Figs. 4 and 5 for an aspect ratio of 3 and a wind angle α_w of 30E and 60E, respectively. The geometric angle α_g for such a rectangle is 71.6E. Note that each curve consists of three separate curves for each of the three regions I, II, and III. The inner linear region, denoted by dashed lines, matches so smoothly with the outer curves that they appear to form a single and continuous curve. Note also that as the wind angle increases from 30E to 60E, the relative size of the linear region increases.

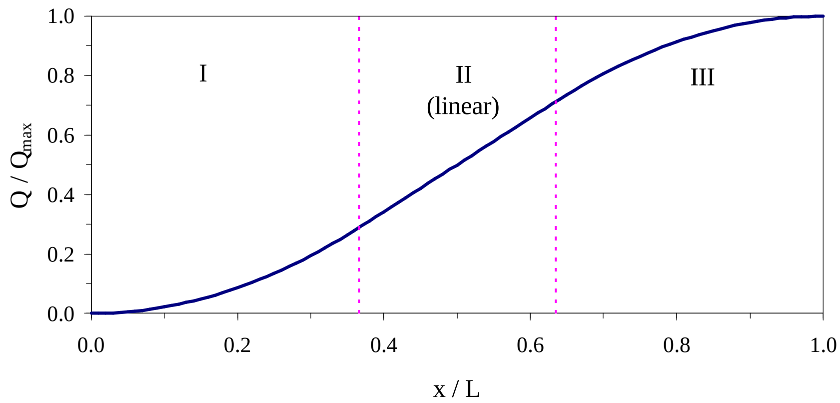

Figure 4. The variation of mass flow across a rectangular source with aspect ratio 3 and wind angle 30E.

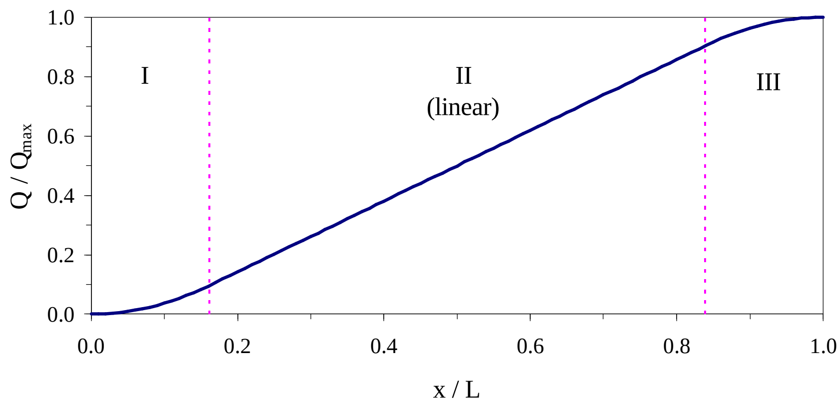

Figure 5. The variation of mass flow across a rectangular source with aspect ratio 3 and wind angle 60E.

Circular or Elliptical Sources

Notation for flow across a circular or elliptical area source is shown in Fig. 6. Originally I had intended to obtain solutions for all wind angles but this task proved more difficult than first imagined. Analytical solutions were obtained for the more limited conditions of α_w = 0E or 90E. In either case, $L(\alpha_w)$ is simply the wind-aligned source width. The maximum crosswind width may be expressed as $\beta L(\alpha_w)$, where β is the ratio of the crosswind and wind-aligned dimensions. If $\beta = 1$ then the shape is circular, if $0 < \beta < 1$ then the major axis is aligned with the wind, and if $\beta > 1$ then the minor axis is aligned with the wind.

The crosswind width function for an ellipse is

$$\frac{S(x)}{L} = 2\,\beta \left[\frac{x}{L} \left(1 - \frac{x}{L} \right) \right]^{½} \qquad (33)$$

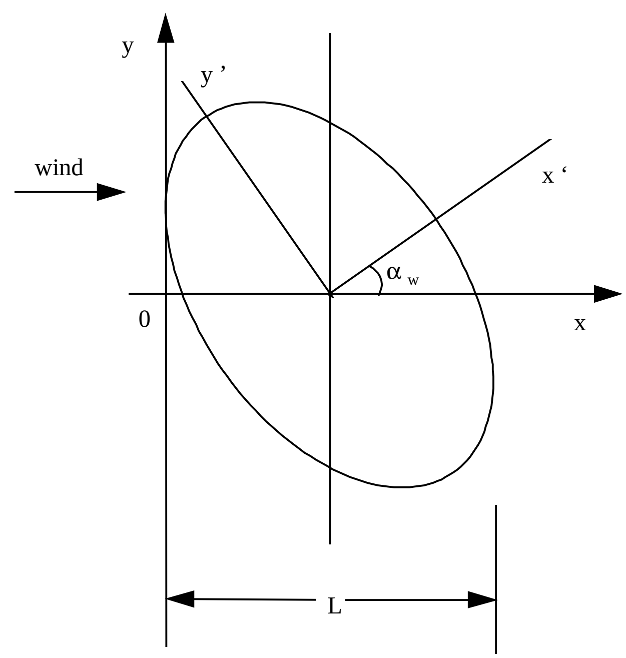

Figure 6. Definition sketch for an elliptical area source.

Substituting Eq. (33) into Eq. (6) and integrating yields:

$$\frac{Q(x)}{Q_{max}} = \frac{2}{\pi}\left(2\frac{x}{L} - 1\right)\left(\frac{x}{L}\left(1 - \frac{x}{L}\right)\right)^{1/2} + \frac{1}{\pi}\sin^{-1}\left(2\frac{x}{L} - 1\right) + \frac{1}{2} \qquad (34)$$

Equation (34), shown graphically in Fig. 7, is a smooth and slightly *S*-shaped curve which rises from zero at the leading edge and reaches a maximum value at the downwind edge of the source. The maximum value of the horizontal mass flow Q_{max} at the downwind edge is equal to the rate of vertical mass transfer from the entire source.

Note that β does not appear explicitly in Eq. (34) when written in dimensionless form. Thus, the single curve plotted in Fig. 7 describes the variation of mass flow across both circular and elliptical sources. The only restriction is that for elliptical sources we are limited to winds that align with either the major or minor axes.

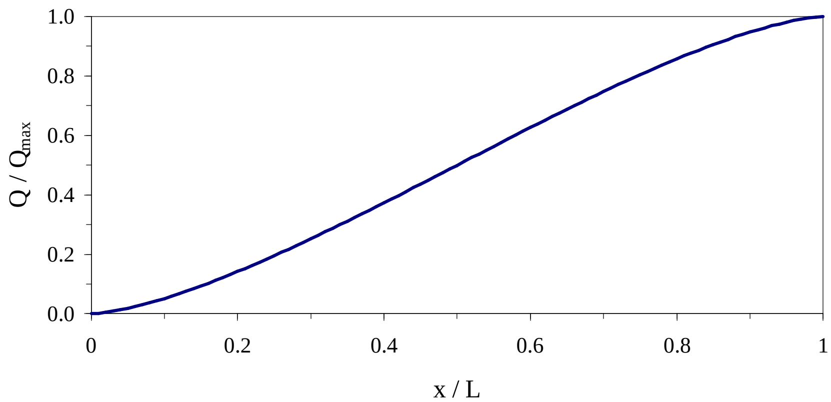

Figure 7. Variation of mass flow across a circular or elliptical source (α_w = 0**E** or 90**E**).

Conclusions

The variation of mass flow across rectangular and elliptical area sources is derived from the basic principle of conservation of mass. Solutions are restricted to conditions of steady and uniform vertical flux of mass from the source and a steady wind direction. However, under these restrictions, analytical solutions are obtained.

Solutions for rectangular sources are generally not continuous functions and must be split into separate solutions according to the wind angle and the resulting distinct geometrical regions within the rectangular shape. Solutions for elliptical area sources are continuous.

References

Chrysikopoulos, C. V., L. M. Hildemann & P. V. Roberts. 1992. A three-dimensional steady-state atmospheric dispersion-deposition model for emissions from a ground-level area source. *Atmospheric Environment.* **26A**, 747-757.

Crane, G., H. A. Panofsky & O. Zeman. 1977. A model for dispersion from area sources in convective turbulence. *Atmospheric Environment* **11**, 893-900.

Petersen, W. B. 1978. **User's Guide for PAL: A Gaussian-Plume Algorithm for Point, Area, and Line Sources.** EPA-600/4-78-013, U.S. Environmental Protection Agency, Research Triangle Park, NC, 155 pp.

Schmid, H. P. 1994. Source areas for scalars and scalar fluxes. *Boundary-Layer Meteorol.* **67**:293-318.

Snyder, W. H. 1991. **Wind-Tunnel Simulation of Dispersion from Superfund Area Sources**. Fluid Modeling Facility Internal Report, U.S. Environmental Protection Agency, Research Triangle Park, NC.

U.S. Environmental Protection Agency. 1989. **Review and Evaluation of Area Source Dispersion Algorithms for Emission Sources at Superfund Sites**. EPA-450/4-89-020. U.S. Environmental Protection Agency, Research Triangle Park, NC.

U.S. Environmental Protection Agency. 1992a. **Comparison of a Revised Area Source Algorithm for the Industrial Source Complex Short Term Model and Wind Tunnel Data.** EPA-454/R-92-014, U.S. Environmental Protection Agency, Research Triangle Park, NC, 174 pp.

U.S. Environmental Protection Agency. 1992b. **User's Guide for the Industrial Source Complex (ISC2) Dispersion Models, Volume I - User Instructions.** EPA-450/4-92-008a, U.S. Environmental Protection Agency, Research Triangle Park, NC.